

# *Final Exam*

## Instructions

- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books.
- You **may** use a simple scientific calculator. **No** graphing or programmable calculators.
- **Take your time. Answer each question completely. Check your answers.**
- *For full credit, show all your work and reasoning.*

**Good Luck!!!**

NAME: \_\_\_\_\_

Problem	Score
1	/8
2	/10
3	/10
4	/8
5	/8
6	/10
Total	/50

1. (8 pts) A firm's production function is given by

$$Q = 20K^{3/5}L^{2/5},$$

where  $Q$  is the firm's monthly output,  $K$  is the firm's monthly capital input and  $L$  is the firm's monthly labor input. Each unit of capital input costs the firm \$1000 and each unit of labor input costs the firm \$600.

Find the levels of capital and labor input that the firm should use to *minimize* the cost of producing  $Q_0 = 2000$  units of output. What is the firm's minimum cost?

Be sure to explain how you know that your answer yields the *absolute* minimum cost of producing 2000 units.



2. The demand equation for a monopolistic firm's product is given by

$$p = 54 - 0.4q,$$

where  $p$  is the price per unit that the firm charges (in dollars) and  $q$  is the daily demand for the firm's product. The firm's daily cost function is

$$c = 10q + 80,$$

where  $c$  is also measured in dollars.

- (a) (8 pts) Find the price that the firm should set to maximize its daily profit, the profit-maximizing output and the maximum daily profit. Be sure to *explain* how you know that the profit you found is the firm's absolute maximum.

- (b) (2 pts) The local government imposes a \$1.60 per unit production tax on the firm. How much of this tax should the firm pass on to consumers (in the form of a price increase) to maximize their profit under the new tax? Show your work.

3. Consider the function  $f(x) = \frac{4x + 3}{x^2 + 1}$ .
- (a) (8 pts) Find the critical points of  $f(x)$  and classify the critical values as relative minima, relative maxima or neither. You may use either the first derivative test or the second derivative test—your choice.
- (b) (2 pts) Does  $f(x)$  attain an *absolute* maximum value on the interval  $(0, \infty)$ ? Justify your answer.



4. The demand equation for the monopolistic firm ACME Widgets is given by

$$q = 1.2(600 - 4p)^{3/2},$$

where  $p$  is the price of a widget and  $q$  is weekly demand for widgets.

- (a) (4 pts) Find the demand,  $q$ , and price-elasticity of demand for widgets,  $\eta_{q/p}$  when  $p = 125$ .

- (b) (2 pts) Use your answer to part **a.** to find the approximate *percentage* change in demand if ACME *lowers* the price of a widget from  $p = 125$  to  $p = 122.5$ .



(c) (2 pts) What is ACME's marginal revenue when  $p = 125$ ? Justify your answer briefly.

5. ACME Widgets' short-term production function is given by

$$q = 75(5l - 6)^{2/3},$$

where  $q$  is ACME's weekly output (which is equal to the weekly demand for their product) and  $l$  is ACME's labor input, measured in 40-hour work weeks. E.g., if  $l = 10$ , then ACME's workers are working for a total of  $10 \times 40 = 400$  hours a week.

The demand equation for ACME's product is given in the previous problem.

- (a) (4 pts) What is ACME's output and marginal product,  $dq/dl$ , when  $l = 14$ ?
- (b) (2 pts) Use your answers to part (a), above, and problem 4(c) to find ACME's marginal revenue product  $dr/dl$  when  $l = 14$ . (You may assume that the demand for the firm's product is equal to their output.)

- (c) (2 pts) If ACME hires a new employee to work 20 hours a week, what is the approximate effect on the firm's revenue?

6. The consumption function for a small nation is given by

$$C = \frac{8Y^2 + 25}{9Y + 5},$$

where  $Y$  is the nation's annual income and  $C$  is the nation's annual consumption, both measured in billions of dollars.

- (a) (6 pts) What are the nation's marginal propensities to consume and to save when its annual income is \$5 billion?

- (b) (2 pts) Use your answer to part **a.** to estimate the changes in savings and consumption when the nation's income increases from \$5 billion to \$5.2 billion.

- (c) (2 pts) Compute the limit  $\lim_{Y \rightarrow \infty} \frac{dC}{dY}$  and interpret your answer in economic terms.

