1. Use the differentiation formulæ (shortcuts) to find the derivatives of the functions below.
Clean up your answers.

(a) (2 pts) $f(x) = 3x^4 + 3x^2 + 5x - 2;\quad f'(x) = 12x^3 + 6x + 5$

(b) (2 pts) $v = 3\sqrt{t} + \dfrac{2}{t^3};\quad \dfrac{dv}{dt} = \dfrac{d}{dt}\left(3t^{1/2} + 2t^{-3}\right) = 3 \cdot \dfrac{1}{2}t^{-1/2} + 2 \cdot (-3)t^{-4} = \dfrac{3}{2}t^{-1/2} - 6t^{-4}$

(c) (2 pts) $y = \dfrac{2x^2+3}{5x+1};\quad y' = \dfrac{4x(5x+1) - 5(2x^2+3)}{(5x+1)^2} = \dfrac{10x^2 + 4x - 15}{(5x+1)^2}$

2. (4 pts) Use the ***definition*** to compute the derivative of $g(x) = 2x^2 + 3$.
Show your work *and use the 'limit' notation correctly.*

$$\begin{aligned}
g'(x) &= \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \\
&= \lim_{h\to 0} \frac{2(x+h)^2 + 3 - (2x^2+3)}{h} \\
&= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3} - \cancel{2x^2} - \cancel{3}}{h} \\
&= \lim_{h\to 0} \frac{4x\cancel{h} + 2h^{\cancel{2}}}{\cancel{h}} \\
&= \lim_{h\to 0} 4x + 2h \\
&= 4x
\end{aligned}$$

3. (5 pts) A firm's marginal cost function is given by

$$\frac{dc}{dq} = \sqrt{5q^2 + 4},$$

where cost (c) is measured in \$1000s and output (q) is measured in 100s of units. By approximately how much will the firm's cost increase if its output increases from 300 units to 340 units?
Show your work*, pay attention to the units, and express your answer as a dollar amount.*

Use linear approximation: $\Delta c \approx \dfrac{dc}{dq} \cdot \Delta q$.
When output is 300 units, then $q = 3$ (because q is measured in 100s of units) and by the same reasoning, if output increases to 340, then $\Delta q = \frac{40}{100} = 0.4$.
It follows that

$$\Delta c \approx \left.\frac{dc}{dq}\right|_{q=3} \cdot \Delta q = \sqrt{5 \cdot 3^2 + 4} \cdot (0.4) = 2.8.$$

Finally, since cost is measured in 1000s of dollars, the approximate change in cost will be about \$2800.