1. Find the derivatives (using the formulas, not the definition).

(a) (2 pts) $f(x) = x^2 \ln(3x+2)$; $\quad f'(x) = 2x\ln(3x+2) + x^2 \cdot \dfrac{3}{3x+2} = 2x\ln(3x+2) + \dfrac{3x^2}{3x+2}$

(b) (2 pts) $y = 10e^{t^2+4}$; $\quad \dfrac{dy}{dt} = 10e^{t^2+4} \cdot (2t) = 20te^{t^2+4}$

(c) (2 pts) $h(x) = \sqrt{3\ln x + 5}$; $\quad h'(x) = \dfrac{1}{2}(3\ln x + 5)^{-1/2} \cdot \dfrac{3}{x} = \dfrac{3}{2x}(3\ln x + 5)^{-1/2} \qquad \left(= \dfrac{3}{2x\sqrt{3\ln x + 5}}\right)$

2. (4 pts) Find the ***quadratic*** Taylor polynomial for the function $f(x) = \sqrt{x}$ centered at $x_0 = 4$, and use this polynomial to approximate $\sqrt{4.5}$.

Differentiate (twice): $f(x) = x^{1/2} \implies f'(x) = \dfrac{1}{2}x^{-1/2} \implies f''(x) = -\dfrac{1}{4}x^{-3/2}$

Evaluate at $x_0 = 4$: $f(4) = 2$, $f'(4) = \dfrac{1}{2} \cdot \dfrac{1}{4^{1/2}} = \dfrac{1}{4}$ and $f''(4) = -\dfrac{1}{4} \cdot \dfrac{1}{4^{3/2}} = -\dfrac{1}{32}$.

Build the Taylor polynomial: $T_2(x) = f(x_0) + f'(x_0)(x - x_0) + \dfrac{f''(x_0)}{2}(x - x_0)^2$, so in this case

$$T_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2.$$

Approximate:

$$\sqrt{4.5} = f(4.5) \approx T_2(4.5) = 2 + \frac{1}{4}(4.5-4) - \frac{1}{64}(4.5-4)^2 = 2 + \frac{1}{4}\cdot\frac{1}{2} - \frac{1}{64}\cdot\frac{1}{4} = \frac{543}{256} \qquad (= 2.12109375)$$

3. The demand equation for a monopolist's product is $q = 10\sqrt{400 - 0.8p}$

(a) (2 pts) Find the price-elasticity of demand for this good ***as a function of the price*** p.

$$\begin{aligned}\eta_{q/p} = \frac{dq}{dp} \cdot \frac{p}{q} &= \overbrace{\left[10 \cdot \frac{1}{2}(400 - 0.8p)^{-1/2} \cdot (-0.8)\right]}^{dq/dp} \cdot \frac{p}{10\sqrt{400 - 0.8p}} \\ &= \frac{-4}{(400-0.8p)^{1/2}} \cdot \frac{p}{10(400-0.8p)^{1/2}} \\ &= \frac{-4p}{10(400-0.8p)} = -\frac{p}{1000-2p}\end{aligned}$$

(b) (1 pt) What is the price elasticity of demand when $p = 200$?

$$\eta_{q/p}\Big|_{p=200} = -\frac{200}{1000-400} = -\frac{1}{3}$$

(c) (2 pts) Use your answer to (b) to find the firm's *marginal revenue* when $p = 200$.

Use the formula

$$\frac{dr}{dq} = p\left(1 + \frac{1}{\eta}\right)$$

So in this case

$$\frac{dr}{dq}\Big|_{p=200} = 200\left(1 + \frac{1}{-1/3}\right) = 200(1-3) = -400$$