1. (6 pts) Find the critical points and critical values of the function $g(t) = 3t^2e^{-0.5t}$.

Differentiate: $g'(t) = 6te^{-0.5t} + 3t^2e^{-0.5t} \cdot (-0.5) = 3te^{-0.5t}(2 - 1.5t)$

Critical points: $g'(t) = 0 \implies t = 0$ or $t = 4/3$.

Critical values: $g(0) = 0$ and $g(4/3) = \dfrac{16}{3}e^{-2/3} \approx 2.738$.

2. (4 pts) Classify the critical values you found above as relative minima, relative maxima or neither.

First derivative test:

$g'(-1) = -3e^{1/2}(2+1.5) < 0$ and $g'(1) = 3e^{-1/2}(2-1.5) > 0$, so $g(0) = 0$ is a relative minimum value. (In fact, it is the absolute minimum value of $g(t)$ on the entire real line. Why?)

$g'(1) > 0$ (see above) and $g'(2) = 12e^{-1}(2-3) < 0$, so $g(4/3) \approx 2.738$ is a relative maximum value. (Can you show that it is the absolute maximum value of $g(t)$ in $(0, \infty)$?)

Or ... second derivative test:

Rewrite $g'(t) = 3te^{-0.5t}(2 - 1.5t) = e^{-0.5t}(6t - 4.5t^2)$, making it easier to find $g''(t)$:

$$g''(t) = -0.5e^{-0.5t}(6t - 4.5t^2) + e^{-0.5t}(6 - 9t) = e^{-0.5t}(2.25t^2 - 12t + 6)$$

And now:

$$g''(0) = e^0 \cdot 6 = 6 > 0,$$

so $g(0) = 0$ is a relative minimum value.

$$g''(4/3) = e^{-2/3}(2.25 \cdot (16/9) - 12 \cdot (4/3) + 6) = e^{-2/3}(-6) < 0$$

so $g(4/3) \approx 2.738$ is a relative maximum value.

3. (5 pts) Find the **absolute** maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 24x + 2$ in the interval $[0, 6]$.

Critical points:

$$f'(x) = 0 \implies 6x^2 - 30x + 24 = 0 \implies x^2 - 5x + 4 = 0 \implies (x-1)(x-4) = 0$$

so the critical points are $x_1 = 1$ and $x_2 = 4$.

Evaluate the function: We evaluate $f(x)$ at the points $0, 1, 4$ and 6 (the endpoints of the ***closed*** interval $[0, 6]$ and the critical points in the interval):

$$f(0) = 2, \quad f(1) = 13, \quad f(4) = -14 \text{ and } f(6) = 38$$

so $f(4) = -14$ is the absolute minimum value in $[0, 6]$ and $f(6) = 38$ is the absolute maximum value there.