

Limits, Continuity and Differentiation

- Without consulting your notes or book, give the mathematical definitions for the two bold-faced concepts below. You should use *limits* in both definitions.
 - The function $f(x)$ is **continuous** at the point $x = a$.
 - The function $f(x)$ is **differentiable** at the point $x = a$.
- Find the points of **discontinuity** of the function $f(x) = \frac{x^2 + 1}{x^2 - 4x + 3}$, and use one-sided limits to describe how the function behaves near these points.
- What can you say about $\lim_{x \rightarrow 0} \frac{x}{|x|}$?
- Give an example of a function that is **continuous** at the point $x = 0$, but is **not differentiable** at $x = 0$. Justify your answer.
- For each of the functions $f(x)$ given below, simplify the expression $\frac{f(x+h) - f(x)}{h}$. Express your answer in terms of x and h .

(a) $f(x) = x^2$	(c) $f(x) = \sqrt{x}$
(b) $f(x) = x^3$	(d) $f(x) = \frac{1}{x}$
- For the functions and points given below, *use the definition of the derivative* to find the slope of the graph of $y = f(x)$ at the point x_0 , and the equation of the tangent line to the graph $y = f(x)$ at the point $(x_0, f(x_0))$.

(a) $f(x) = x^2$; $x_0 = 1$.	(c) $f(x) = \frac{2}{x}$; $x_0 = 3$.
(b) $f(x) = 3x^2 + 2x - 1$; $x_0 = 2$.	(d) $f(x) = 3\sqrt{x}$; $x_0 = 4$.
- Use the **rules of differentiation** to compute the derivatives of the functions below.

(a) $f(x) = 3x^2 - 2x + 5$; $f'(x) =$	(c) $g(t) = \frac{3x + 4}{x^2 + 5}$; $g'(t) =$
(b) $y = 2\sqrt{x} - \frac{3}{x^2}$, $\frac{dy}{dx} =$	(d) $h(x) = (2x + 5)(x^2 + 2x + 3)$; $h'(x) =$
- Find the derivative of $f(x) = \sqrt[3]{x}$ at the point $x = 8$.
 - Use your answer to (a) and *linear approximation* to estimate $\sqrt[3]{9}$.
 - Use the same ideas to estimate $\sqrt{102}$.

9. A firm's marginal revenue function is given by

$$\frac{dr}{dq} = 0.7q - 0.05q^2,$$

where revenue r is measured in \$1000s and output q is measured in 100s of units. By approximately how much will the firm's revenue change if output increases from 1000 units to 1050 units?