AMS 11A **Study Guide 3** ECON 11A

More differentiation; Elasticity; Taylor polynomials

1. Compute the derivatives of the functions below

a. $f(x) = \sqrt[3]{x^3 - 3x^2 + 1}$

b. $y = \ln(5x + 3)$

c. $s = e^{0.05t}$

d. $g(x) = \log_5 x$

e. $h(u) = 3^u$

f. $f(x) = 3x^2 e^x$

g. $y = \ln(x^2 + 2x + 3)$

h. $g(t) = e^{t^3 - 2t + 1}$

i. $k(u) = \dfrac{u \ln u}{2u + 1}$

2. The consumption function for a small country is given by

$$C = \ln\left(\frac{e^{0.95Y}}{e^{0.2Y} + 5}\right),$$

where Y is national income, measured in \$ billions.

a. How much is consumed when $Y = 10$?

b. What is the marginal propensity to consume when $Y = 10$?

c. By approximately how much will consumption increase if income increases from \$10 billion to \$10.4 billion? By how much will savings increase?

d. Compute the limit $\lim\limits_{Y \to \infty} \dfrac{dC}{dY}$, and interpret the result.

3. The *marginal revenue* function of a monopolistic firm is given by

$$\frac{dr}{dq} = \sqrt{250 - q},$$

where revenue is measured in \$1000s per month and the firm's output q is measured in 1000s of units per month. The firm's production function is

$$q = 30(4m - 15)^{1/3},$$

where m is the firm's labor input measured in 40-hour work weeks (e.g., if $m = 5$, then the firm's employees are working a combined 200 hours a week and if $m = 17.2$, then the firm's employees are working a combined 4288 hours a week). The firm's current labor input is $m = 35$.

a. Find the firm's output and *marginal product of labor* at the current level of labor input.

b. Find the firm's *marginal revenue product* at the current level of labor input.

c. The firm decides to increase its labor force and hires an additional part-time laborer, to work 10 hours a week. The total monthly expense to the firm for the new employee is \$2450.00. Use your answer to part **b.** to estimate the change in the firm's monthly profit.

4. Find the *labor-elasticity of output* for the firm in the problem above when $m = 35$. Use your answer to estimate the *percentage* change in output, if the firm increases its labor input by 30 hours a week.

5. The demand equation for a monopolist's product is $p = 250 - 0.2q$.

a. Find the price-elasticity of demand (as a function of q).

b. What is the price elasticity of demand when $p = \$50$? Is demand elastic, inelastic, or does demand have unit elasticity at this point?

c. Suppose that the price is lowered (from \$50) to \$49.25. Use your answer to part b. to estimate the *percentage* change in demand.

d. What effect will this change in price have on the firm's revenue? Be as precise as you can, and explain your answer.

6. Find the cubic Taylor polynomial for the function $f(x) = \sqrt{x}$, centered at the point $x_0 = 25$. Use this polynomial to estimate $\sqrt{26}$.