Study Guide 5

Applied optimization and curve sketching

1. Sketch a graph of the function

$$g(x) = \frac{2x+3}{x^2+2}.$$

Your sketch should clearly indicate: x and y intercepts; critical points and relative extreme values; horizontal asymptotes (i.e.,limits at $\pm\infty$); intervals where the function is increasing or decreasing; intervals where the graph is concave up or down; inflection points.

Hint: to find the intervals where $g(x)$ is concave up and where it is concave down, it will be helpful to know that the roots of the equation $2x^3 + 9x^2 - 12x - 6 = 0$ are (approximately) $r_1 \approx -5.49$, $r_2 \approx -0.4$ and $r_3 \approx 1.38$.

2. The demand equation for a monopolistic firm's product is given by $p = 830 - 2q - 0.05q^2$, where p is the price of the firm's product and q is weekly demand. The *constant* marginal cost of the firm's output is \$50 and the firm's weekly fixed cost is \$5000. Find the price the firm should set to maximize its weekly profit, as well as the corresponding output level and the max profit. Justify your claim that the price you found yields the *absolute* maximum profit.

3. A firm's cost function is given by $c = 0.02q^2 + 20q + 800$. Find the level of output that minimizes the firm's *average* cost.

4. Farmer Jones wants to build a 4800 square foot rectangular enclosure for her vegetable garden. The enclosure will be surrounded by grade A fencing that costs \$12.00 per linear foot, and the interior of the enclosure will be subdivided into 5 equal parts using grade B fencing that costs \$8.00 per linear foot, (see Figure 1 below). What should the dimensions of the enclosure be to minimize the total cost of the fencing? What will the minimal cost be?

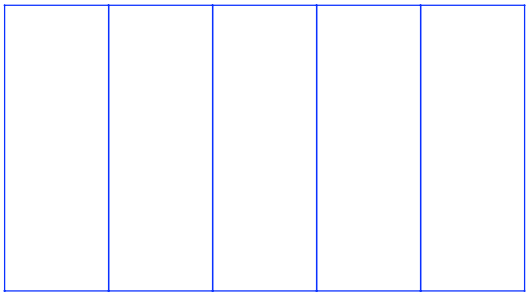

Figure 1: Farmer Jones' vegetable garden.

5. The *present value* of a bottle of fine cognac is given by $V(t) = 90t^{2/3}e^{-0.05t}$, where t is measured in years and $V(t)$ is measured in dollars. How many years should an investor hold the bottle of cognac before selling it, to maximize its present value? Justify your answer.

6. The production function for ACME Widgets is $q = 20k^{0.6}l^{0.5}$, where q is annual output, measured in 1000s of widgets, k is the capital input and l is labor input. The cost per unit of capital input is \$1000 and the cost per unit of labor input is \$5000.

a. Find the levels of capital and labor input that ***maximize*** output, given that ACME's annual production budget is $B = \$1.1$ million. *Justify your claim* that you found the *absolute maximum*. What is ACME's maximum output?

b. What proportion of the total budget is spent on capital input and what proportion is spent on labor input? Do you notice anything interesting about these proportions?

7. A firm's production function is given by $Q = 25k^{3/5}l^{2/5}$, where Q is the firm's annual output, k is the firm's annual capital input and l is the firm's annual labor input. The cost per unit of capital input is \$1,000,000 and the cost per unit of labor input is \$50,000.

a. Find the levels of capital and labor input that the firm should use to ***minimize*** the cost of producing 10000 units. What is the the firm's minimum cost? Justify your claim that cost you found is the absolute minimum.

b. Find the levels of capital and labor input that the firm should use to ***minimize*** the cost of producing q units. Express the optimal input levels and the minimum cost in terms of the output q. Once again, justify your claim that you found the firm's absolute minimum cost to produce q units.