

Final Exam

March 17th, 2015

Instructions

- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books.
- You **may** use a simple scientific calculator. **No** graphing or programmable calculators.
- **Take your time. Answer each question completely. Check your answers.**
- *For full credit, show the steps in your work, and explain what you are doing.*

Good Luck!!!

NAME: _____

Problem	Score
1	/10
2	/8
3	/10
4	/8
5	/10
6	/8
Total	/50

1. The demand equation for a monopolistic firm's product is given by

$$p = 150 - 0.75q$$

and the firm's cost function is

$$c = 0.05q^2 + 14q + 800.$$

- (a) (6 pts) Find the price that the firm should set to maximize its profit and find the maximum profit. Explain how you know that the price you found yields the *absolute* maximum profit.

- (b) (4 pts) The state imposes a production tax on the firm of \$4.80 per unit. By how much should the firm increase the price of its product to maximize its profit under this new condition?

2. (8pts) The Jones Family's utility function is given by

$$U = 8 \ln x + 17 \ln y,$$

where U is their monthly utility, x is the quantity of essential goods that they consume per month and y is the quantity of luxury goods that they consume per month. The average price per unit of essential goods is $p_x = \$10$ and the average cost per unit of luxury goods is $p_y = \$40$.

Find the quantities of essential and luxury goods that the Jones family should consume per month to maximize their utility, given that their monthly budget for these goods is $B = \$5000$. What is their maximum utility? Be sure to justify your claim that the utility you find is the *absolute maximum*.

3. Consider the function $f(x) = \frac{4x - 3}{x^2 + 1}$.

- a. (6 pts) Find the critical points of $f(x)$ and classify the critical values as relative minima, relative maxima or neither.
- b. (2 pt) Does $f(x)$ have an absolute maximum on the interval $(0, \infty)$? Justify your answer.
- c. (2 pts) Does $f(x)$ have an absolute maximum on the interval $(-\infty, \infty)$? Justify your answer.

4. The short-run production function of ACME Widgets Inc. is given by

$$q = 50(5\ell - 8)^{2/3},$$

where q is the number of Widgets ACME produces in a month, and ℓ is ACME's monthly labor input, measured in \$1000's.

- a. (4 pts) Find ACME's *labor-elasticity of output* as a function of ℓ .
- b. (2 pts) What is the labor-elasticity of output when monthly labor input is $\ell = 7$?
- c. (2 pts) Use your answer to **b.** to estimate the *percentage change* in output if ACME increases its labor input from \$7,000.00 to \$7,400.00.

5. ACME Widgets' marginal revenue function is given by

$$\frac{dr}{dq} = \frac{30q + 47}{0.1q^2 + 10},$$

where r is the monthly revenue, measured in \$1000's. ACME's production function is given in the previous problem.

- a. (5 pts) What is ACME's marginal-revenue product when labor input is $\ell = 7$?
- b. (3 pts) What is the approximate change to the firm's revenue if ACME increases its labor input to \$7,400.00?
- c. (2 pts) Assuming that labor input is the only variable that is changing, what is the approximate effect on ACME's *profit*? Explain.

6. The savings function for a small nation is given by

$$S = \frac{2Y^2 + 3}{11Y + 20},$$

where Y is the nation's annual income and S is the nation's annual savings, both measured in billions of dollars.

- a. (4 pts) What are the nation's marginal propensities to consume and to save when its annual income is \$7 billion.
- b. (2 pts) Use your answer to part a. to estimate the changes in consumption and savings when the nation's income increases from \$7 billion to \$7.3 billion.
- c. (2 pts) Compute $\lim_{Y \rightarrow \infty} \frac{dS}{dY}$ and interpret the result in economic terms.

