

# *Final Exam*

*March 15<sup>th</sup>, 2016*

## Instructions

- There are 6 questions worth a total of 48 points. 100%=45 points.
- No notes or books.
- You **may** use a simple scientific calculator. **No** graphing or programmable calculators.
- **Take your time. Answer each question completely. Check your answers.**
- *For full credit, show the steps in your work, and explain what you are doing.*

**Good Luck!!!**

NAME: \_\_\_\_\_

Problem	Score
1	/8
2	/8
3	/8
4	/8
5	/8
6	/8
Total	/45

1. The demand equation for a monopolistic firm's product is given by

$$p = 120 - 0.65q$$

and the firm's cost function is

$$c = 0.05q^2 + 15q + 1000.$$

- (a) (6 pts) Find the price that the firm should set to maximize its profit and find the maximum profit. Explain how you know that the price you found yields the *absolute* maximum profit.
- (b) (2 pts) The state imposes a production tax on the firm of \$4.20 per unit. By how much should the firm increase the price of its product to maximize its profit under this new condition?



2. Consider the function  $f(x) = \frac{2x - 3}{x^2 + 4}$ .

- a. (6 pts) Find the critical points of  $f(x)$  and classify the critical values as relative minima, relative maxima or neither.
- b. (1 pt) Does  $f(x)$  have an absolute maximum on the interval  $(0, \infty)$ ? Justify your answer.
- c. (1 pts) Does  $f(x)$  have an absolute maximum on the interval  $(-\infty, \infty)$ ? Justify your answer.



3. (8 pts) The production function for a certain firm is given by

$$Q = 20K^{0.7}L^{0.3}$$

where  $Q$  is the firm's annual output,  $K$  is the firm's capital input and  $L$  is the firm's labor input. The price per unit of capital is  $p_k = 10000$  and the price per unit of labor is  $p_l = 5000$ .

Find the levels of capital and labor input that **maximize the firm's output** given an annual production budget of  $B = 1,000,000$ , and find the maximal output.

*Justify your claim that the output you found is the firm's absolute maximum.*



4. The short-run production function of ACME Widgets Inc. is given by

$$q = 40(2\ell - 8)^{3/4},$$

where  $q$  is the number of Widgets ACME produces in a month, and  $\ell$  is ACME's monthly labor input, measured in \$1000's.

- a. (4 pts) Find ACME's *labor-elasticity of output* as a function of  $\ell$ .
- b. (2 pts) What is the labor-elasticity of output when monthly labor input is  $\ell = 12$ ?
- c. (2 pts) Use your answer to **b.** to estimate the *percentage change* in output if ACME increases its labor input from \$12,000.00 to \$12,600.00.





5. The consumption function for a small nation is given by

$$C = \frac{9Y^2 + 25}{10Y + 3},$$

where  $Y$  is the nation's annual income and  $C$  is the nation's annual consumption, both measured in billions of dollars.

- a. (4 pts) What are the nation's marginal propensities to consume and to save when its annual income is \$5 billion.
- b. (2 pts) Use your answer to part a. to estimate the changes in consumption and savings when the nation's income increases from \$5 billion to \$5.2 billion.
- c. (2 pts) Compute  $\lim_{Y \rightarrow \infty} \frac{dC}{dY}$  and interpret the result in economic terms.



6. (8 pts) The Smith Family's utility function is given by

$$U = 7 \ln x + 13 \ln y,$$

where  $U$  is their monthly utility,  $x$  is the quantity of essential goods that they consume per month and  $y$  is the quantity of luxury goods that they consume per month. The average price per unit of essential goods is  $p_x = \$10$  and the average cost per unit of luxury goods is  $p_y = \$30$ .

Find the quantities of essential and luxury goods that the Smith family should consume per month to maximize their utility, given that their monthly budget for these goods is  $B = \$3600$ . What is their maximum utility? Be sure to justify your claim that the utility you find is the *absolute maximum*.



